

# Alternating current (AC) theory

## Underpinning Knowledge



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## Waveform characteristics

In electronics there are various waveforms that can be encountered: Sine waves, Square Waves, Rectangular Waves, Triangular Waves, Sawtooth Waveforms and a variety of pulses and spikes.

Each of these waveforms has standard characteristics.

Waveforms which are positive or negative, flowing in one forward direction only not cross the zero axis point. These are called uni-directional Waveforms

Waveforms which alternate between a positive value and a negative value are called Bi-directional waveforms.

These waveforms have the following characteristics:

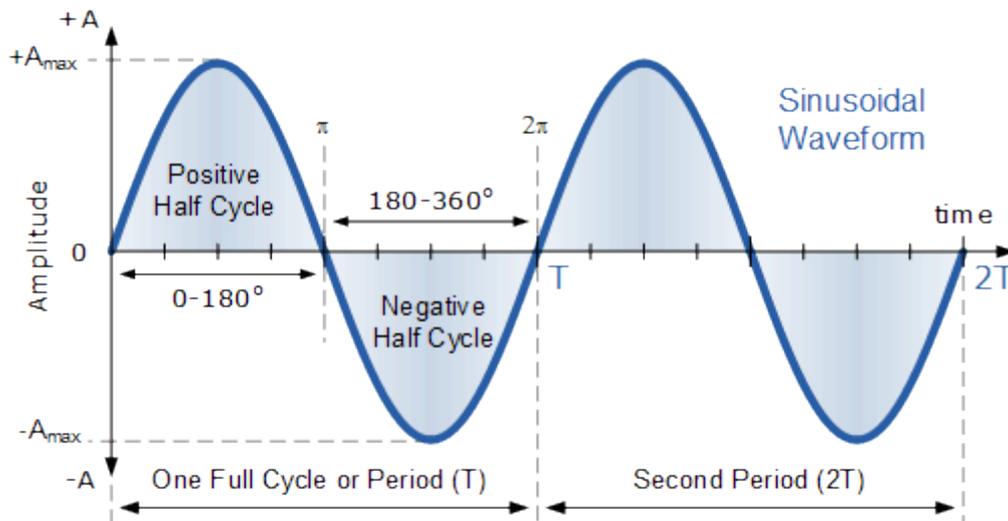
Period: – The length of time in seconds that the waveform takes to repeat itself from start to finish. This value can also be called the *Periodic Time*, (T).

Frequency: – The number of times the waveform repeats itself within a one second time period. Frequency is the reciprocal of the time period, ( $f = 1/T$ ) in *Hertz*, (Hz).

Amplitude: – This is the magnitude or intensity of the signal waveform.

## Sinewaves

**Sine waves** are the most common of all electrical waveforms, changing between its positive and negative values. The time taken for one cycle is called the “periodic time”.



The reciprocal of the period, (  $1/T$  ) gives you the Frequency in **Hertz, (Hz)**. Then Hertz can also be defined as “cycles per second” (cps) and 1Hz is exactly equal to 1 cycle per second.

Both period and frequency are mathematical reciprocals of each other and as the periodic time of the waveform decreases, its frequency increases and vice versa with the relationship between *Periodic time* and *Frequency* given as.

### Relationship between Frequency and Periodic Time

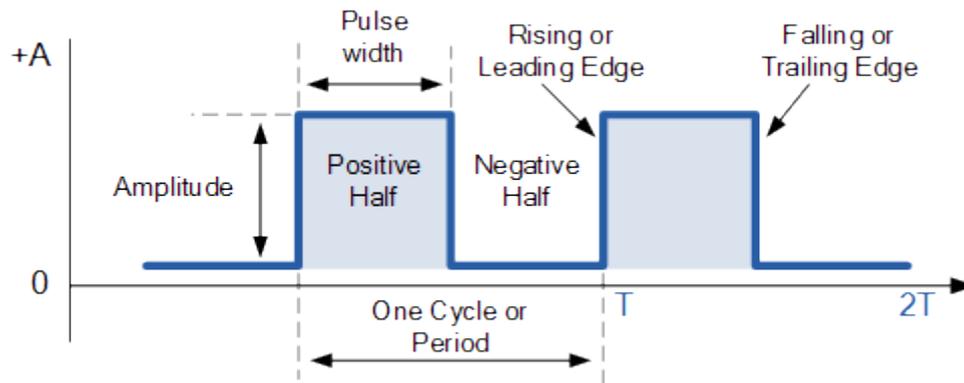
$$\text{Frequency} = \frac{1}{\text{Periodic time}} \quad \text{or} \quad f = \frac{1}{T} \text{ Hz}$$

$$\text{Periodic time} = \frac{1}{\text{Frequency}} \quad \text{or} \quad T = \frac{1}{f} \text{ sec}$$

Where:  $f$  is in Hertz and  $T$  is in Seconds.

## Square Waves

Square Waves have very fast rising and falling edges.

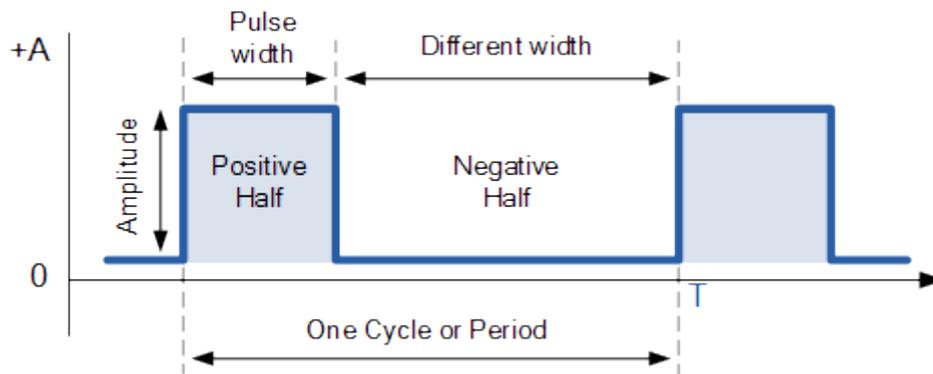


The waveforms are symmetrical where each half of the cycle is identical. The time for the positive and negative pulses is the same. The frequency is equal to the reciprocal of the period,  $(1/T)$  we can define the frequency of a square wave waveform as:

$$\text{Frequency} = \frac{1}{\text{"ON" time} + \text{"OFF" time}}$$

## Rectangular Waveforms

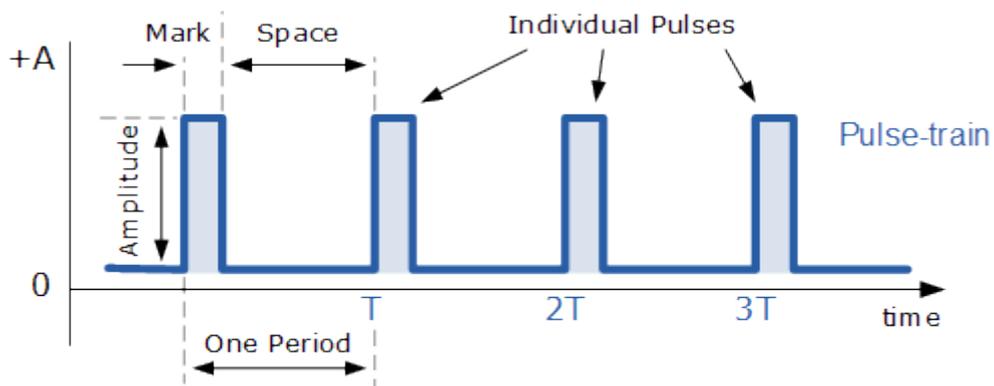
Similar to the square wave but the two pulse widths of the waveform are not the same or “Non-symmetrical”.



These positive and negative pulse widths are sometimes called “Mark” and “Space” respectively, with the ratio of the Mark time to the Space time being known as the “Mark-to-Space” ratio.

$$\text{Frequency} = \frac{1}{\text{Period}} = \frac{1}{10\text{mS} + 30\text{mS}} = 25\text{Hz}$$

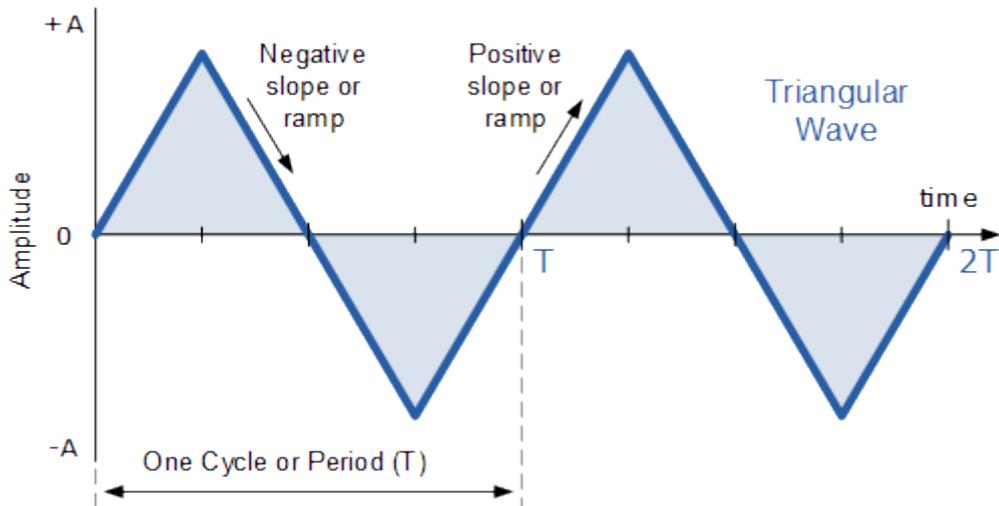
## Pulse Waveform



A **Pulse** waveform has a different Mark-to-Space ratio.

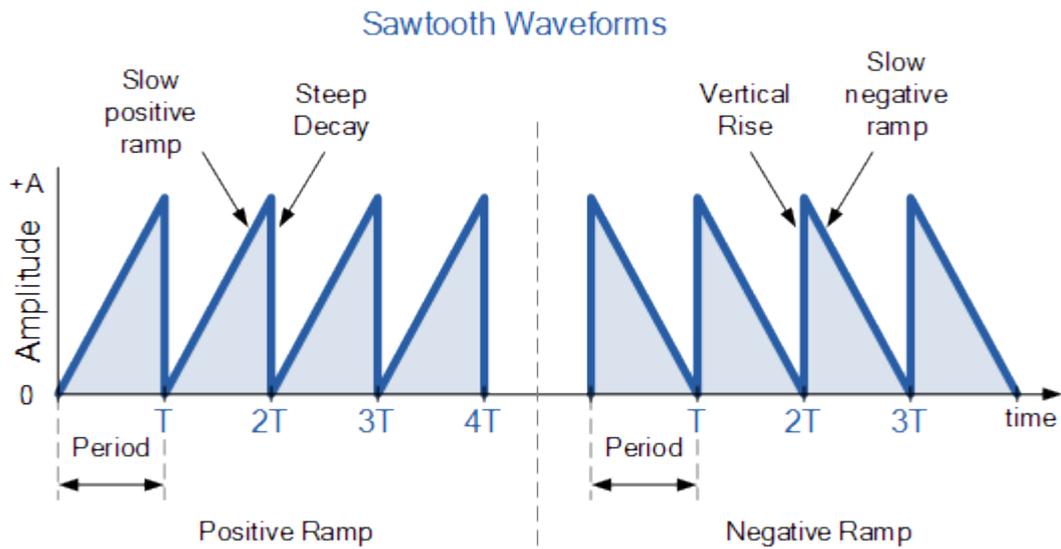
## Triangular Waveforms

These are generally bi-directional non-sinusoidal waveforms that oscillate between a positive and a negative peak value.



## Sawtooth Waveforms

These represent the teeth of a saw blade.



## Worksheet 1

1 If a waveform has the following Periodic times, what is the frequency?

- a) 1ms
- b) 20ms
- c) 35ms
- d) 5us
- e) 560us
- f) 1ns
- g) 33ns
- h) 43ps
- i) 10s
- j) 1 minute

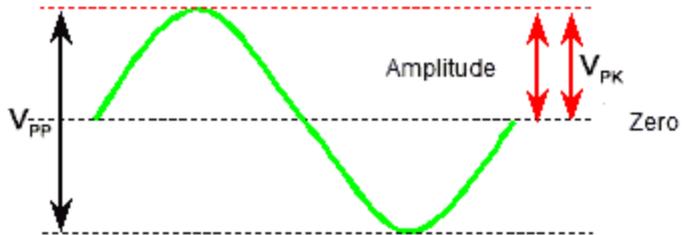
If a waveform has the following frequency, what are the Periodic times?

- a) 10Hz
- b) 560Hz
- c) 12KHz
- d) 120KHz
- e) 1MHz
- f) 33MHz
- g) 100MHz
- h) 4GHz
- i) 0.6Hz
- j) 0.01Hz

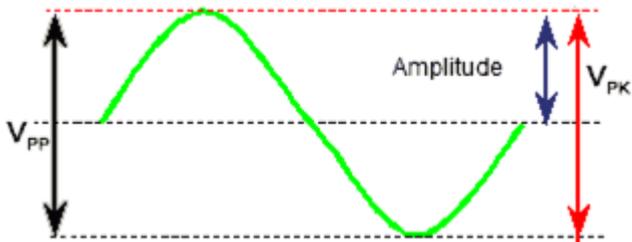
## Practical: Sinewaves using PSPICE

## Peak value

The PEAK value of the waveform is the highest value the wave reaches above a reference value. This is normally from the 0v line. Sometime it is called amplitude.



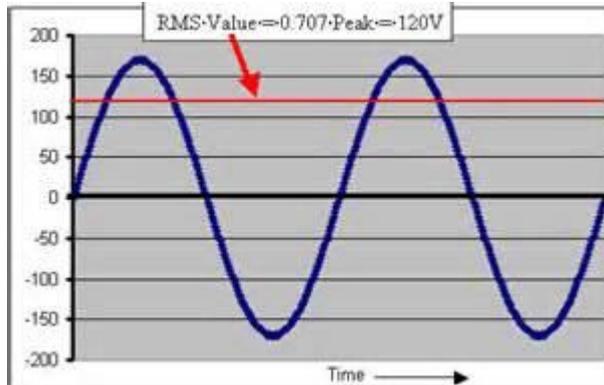
## Peak-to-peak value



The PEAK TO PEAK value is the vertical distance between the top and bottom of the wave.

## Root-mean-square (RMS)

Root-mean-square (RMS) value of an alternating voltage is the equivalent D.C. voltage that can deliver the same amount of energy to a resistor as the A.C. does over a cycle.



The root mean square values of p.d. ( $V_{rms}$ ) and current ( $I_{rms}$ ) represent the effective value of the p.d. and current in an AC circuit.

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

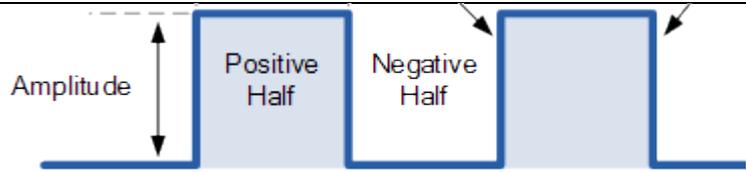
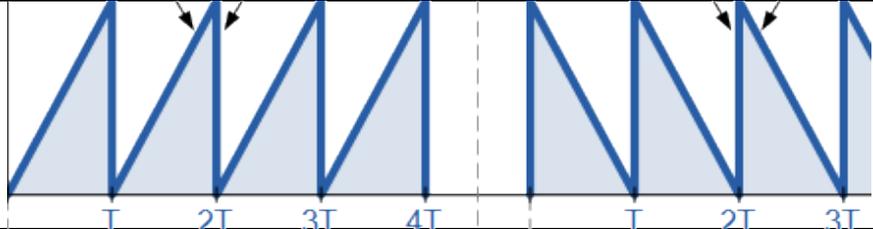
- $V_{rms}$  = root mean square potential difference in volts, V
- $V_o$  = peak voltage in volts, V

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

$I$  = root mean square current in amperes, A

$I_o$  = peak current in amperes, A

### RMS of Non Sinusoidal Waveforms

Waveform		RMS
Squarewave	 <p>The diagram shows a square wave with a period of 2T. The first half-cycle (from 0 to T) is labeled 'Positive Half' and the second half-cycle (from T to 2T) is labeled 'Negative Half'. A vertical double-headed arrow indicates the 'Amplitude' of the positive half-cycle.</p>	$V_{PK}$
Sawtooth	 <p>The diagram shows a sawtooth wave with a period of T. The wave consists of a series of upward-sloping linear ramps followed by a sharp downward reset. The x-axis is marked with T, 2T, 3T, and 4T.</p>	$V_{PK}$ $1/\sqrt{3}$

Task: What are the RMS voltages for the following sinewaves

1. 1 v Pk
2. 4.4 Pk
3. 18.7PK
4. 55.98Pk
5. 339
6. 20v Pk to Pk
7. 56.6v Pk to Pk
8. 0.918 Pk to Pk

## Average value

The AVERAGE value. This is normally taken to mean the average value of only half a cycle of the wave. If the average of the full cycle was taken it would of course be zero, as in a sine wave symmetrical about zero, there are equal excursions above and below the zero line.

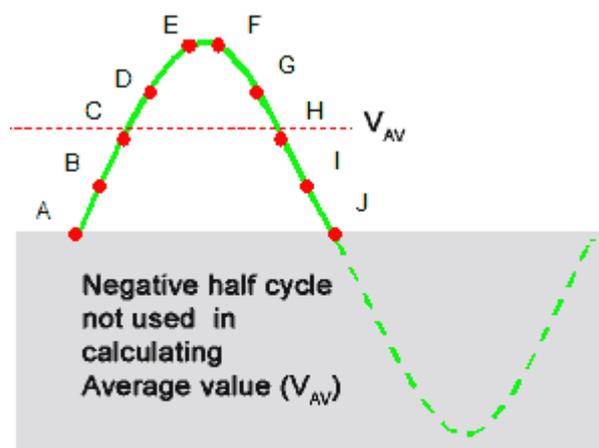
Using only half a cycle, as shown below the average value (voltage or current) is always 0.637 of the peak value of the wave.

$$V_{AV} = V_{PK} \times 0.637$$

or

$$I_{AV} = I_{PK} \times 0.637$$

The average value is the value that usually determines the voltage or current indicated on a test meter. There are however some meters that will read the RMS value, these are called "True RMS meters".



$$V_{AV} \text{ (for the positive half cycle)} = \frac{A+B+C+D+E+F+G+H+I+J}{10}$$

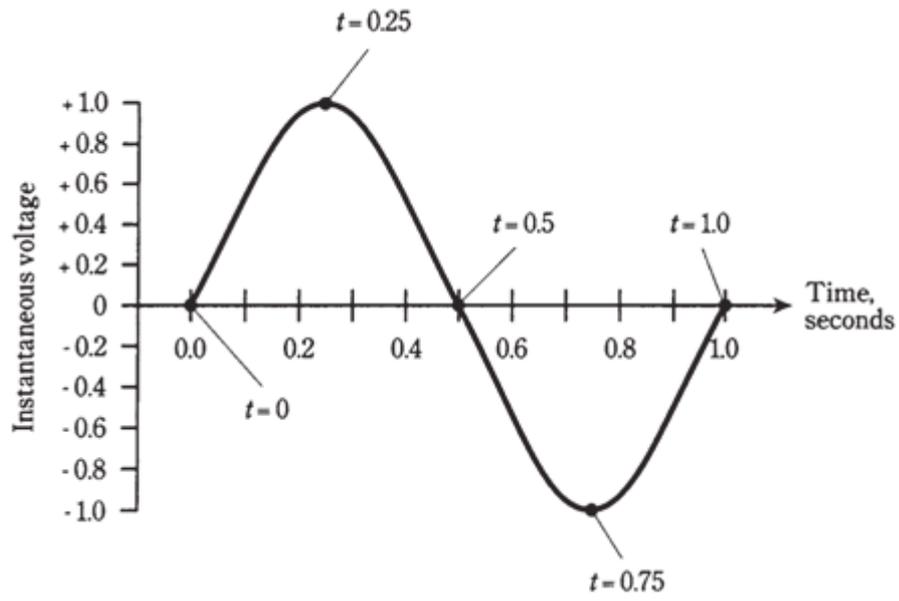
For a pure sine wave  $V_{AV}$  will always be the PEAK value ( $V_{PK}$ )  $\times$  0.637

What are the Average voltages for the following sinewaves

1. 1 v Pk
2. 4.4 Pk
3. 18.7PK
4. 55.98Pk
5. 339
6. 20v Pk to Pk
7. 56.6v Pk to Pk
8. 0.918 Pk to Pk

## Instantaneous

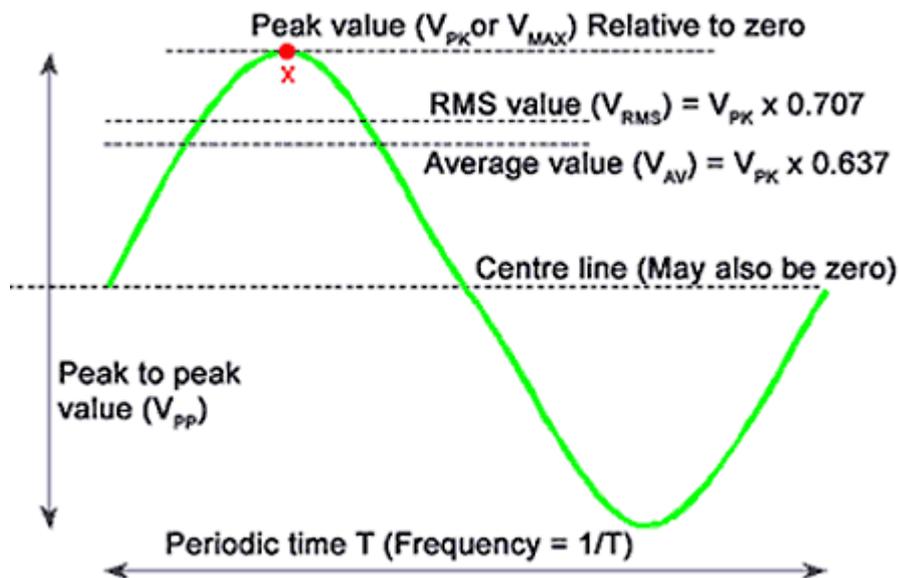
This is the value of a wave at any particular instant of time.



## Form Factor

If  $V_{AV}$  (0.637) is multiplied by 1.11 the answer is 0.707, which is the RMS value. This difference is called the Form Factor of the wave, and the relationship of 1.11 is only true for a perfect sine wave. If the wave is some other shape, either the RMS or the average value (or both) will change, and so will the relationship between them. This is important when measuring AC voltages with a meter as it is the average value that most meters actually measure. However they display the RMS value simply by multiplying the voltage by 1.11. Therefore if the AC wave being measured is not a perfect sine wave the reading will be slightly wrong. If you pay enough money however, you can buy a true RMS meter that actually calculates the RMS value of non-sine waves.

## Waveform summary



## Worksheet 1

If a waveform has the following voltages, what is the RMS voltage?

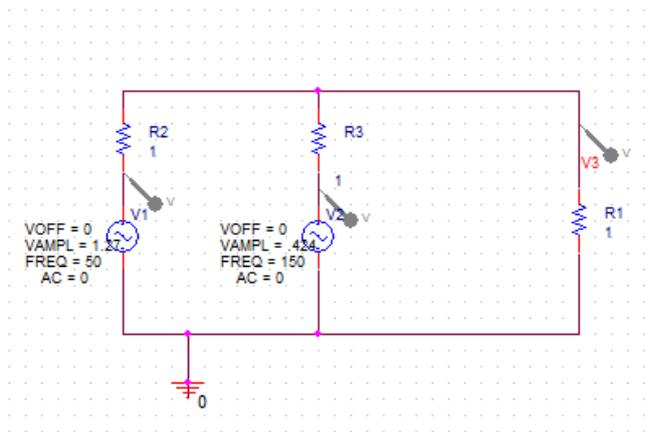
- a) 1v (pk)
- b) 2v (pk to pk)
- c) 35v (pk)
- d) 5v (pk to pk)
- e) 560v(pk)
- f) .11v(pk)
- g) 33v(pk to pk)
- h) 43v(pk)
- i) 10v(pk)
- j) 99v(pk to pk)

## Practical 1

Determination of values using phasor and algebraic representation of alternating quantities

Graphical addition of two sinusoidal voltages

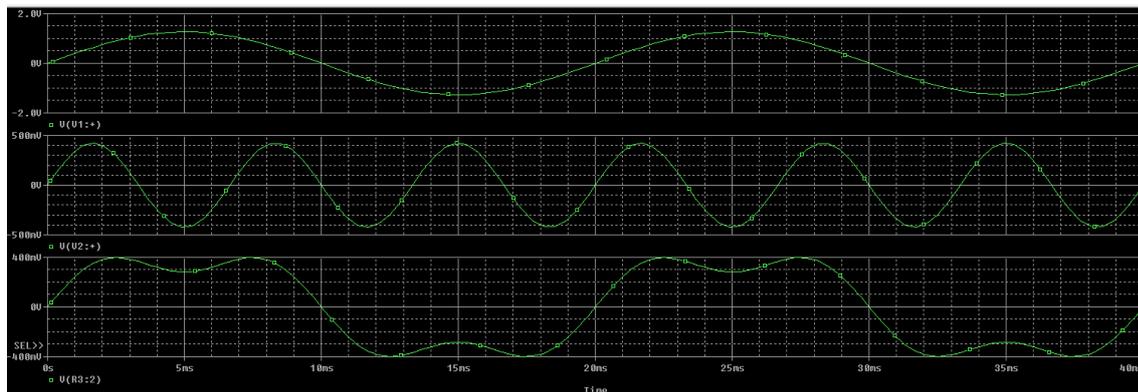
Consider the following circuit



Two waveforms, waveform 1 is 50HZ at 1.27v and the second is 150 Hz at 0.424v are applied to a load resistor.

(See below). If we add these waveforms we get the waveform shown below.

This can be proved using Fouriers



## Task

Complete the following table.

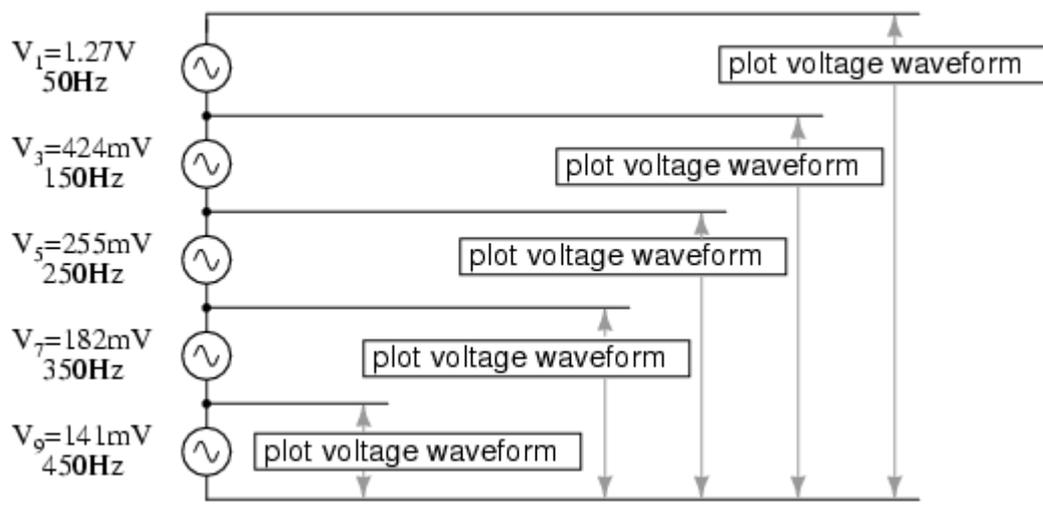
	0°	22.5°	45°	72.5°	90°	112.5°	135°	157.5°	180°	202.5°	225°	247.5°	270°	292.5°	315°	337.5°	360°
Sin x																	
0.33Sin 3x																	
Sin x + 0.33Sin 3x																	
Sin x - 0.33Sin 3x																	

Plot the values of the table above on graph paper.

Development task.

Attempt the following additions.

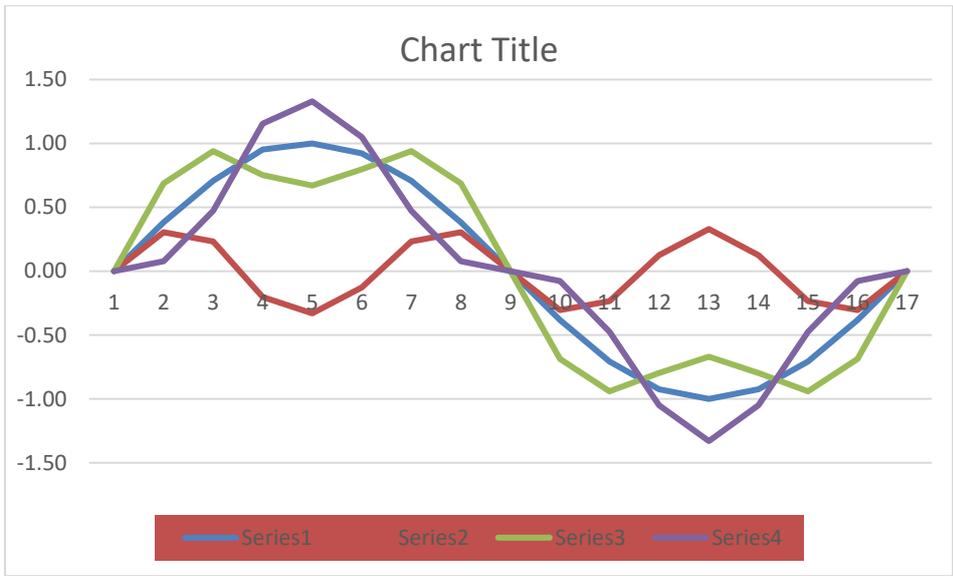
If we consider the following circuit:



## Graphical addition of two sinusoidal voltages using EXCEL

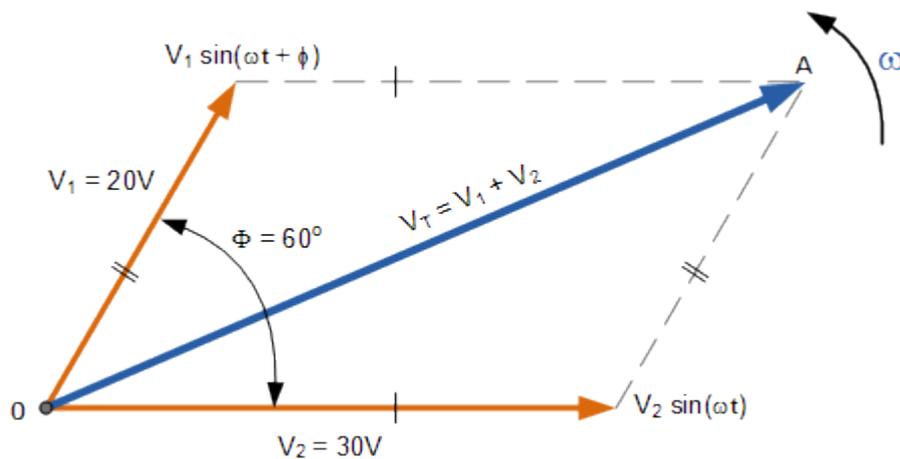
	Angle																
	0	22.5	45	72.5	90	112.5	135	158	180	202.5	225	247.5	270	292.5	315	337.5	360
sin x	0.00	0.38	0.71	0.95	1.00	0.92	0.71	0.38	0.00	-	-	-	-	-	-	-	0.00
0.33 Sin 3x	0.00	0.30	0.23	-	-	-	0.23	0.30	0.00	-	-	0.13	0.33	0.13	-	-	0.00
Sin x+0.33 Sin3x	0.00	0.69	0.94	0.75	0.67	0.80	0.94	0.69	0.00	-	-	-	-	-	-	-	0.00
Sin x - 0.33 Sin3x	0.00	0.08	0.47	1.15	1.33	1.05	0.47	0.08	0.00	-	-	-	-	-	-	-	0.00

Note: Excel uses radians instead of degrees.





## Phasor addition of two sinusoidal voltages



By drawing out the two phasors to scale onto graph paper, their phasor sum  $V_1 + V_2$  can be easily found by measuring the length of the diagonal line, known as the “resultant r-vector”, from the zero point to the intersection of the construction lines 0-A. The downside of this graphical method is that it is time consuming when drawing the phasors to scale. Also, while this graphical method gives an answer which is accurate enough for most purposes, it may produce an error if not drawn accurately or correctly to scale. Then one way to ensure that the correct answer is always obtained is by an analytical method.

Mathematically we can add the two voltages together by firstly finding their “vertical” and “horizontal” directions, and from this we can then calculate both the “vertical” and “horizontal” components for the resultant “r vector”,  $V_T$ . This analytical method which uses the cosine and sine rule to find this resultant value is commonly called the **Rectangular Form**.

### Complex Numbers Example No1

Two vectors are defined as,  $A = 4 + j1$  and  $B = 2 + j3$  respectively. Determine the sum and difference of the two vectors in both rectangular ( $a + jb$ ) form and graphically as an Argand Diagram.

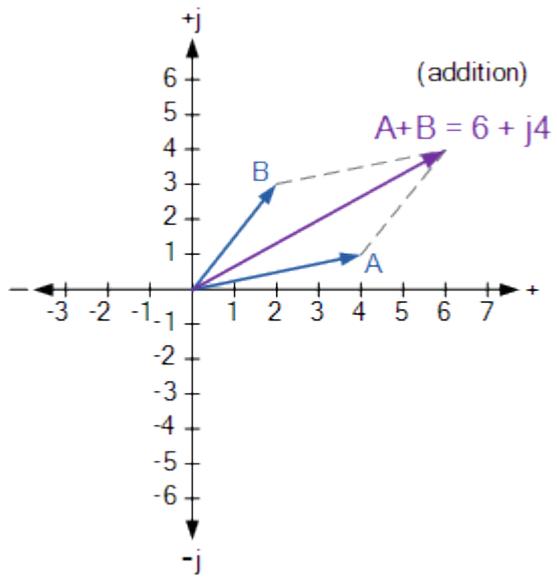
#### Mathematical Addition

$$A + B = (4 + j1) + (2 + j3)$$

$$A + B = (4 + 2) + j(1 + 3)$$

$$A + B = 6 + j4$$

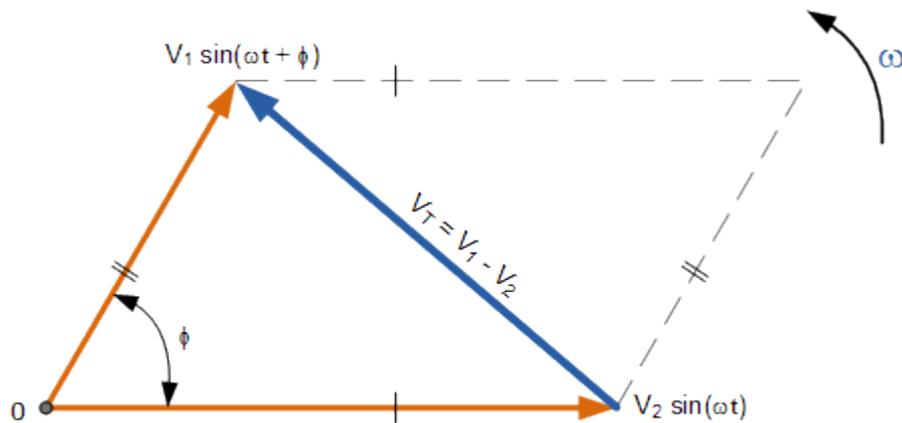
## Graphical Addition



## Phasor Subtraction

Phasor subtraction is very similar to the above rectangular method of addition, except this time the vector  $V_1$  difference is the other diagonal of the parallelogram between the two voltages of  $V_1$  and  $V_2$  as shown.

### Vector Subtraction of two Phasors



This time instead of “adding” together both the horizontal and vertical components we take them away, subtraction.

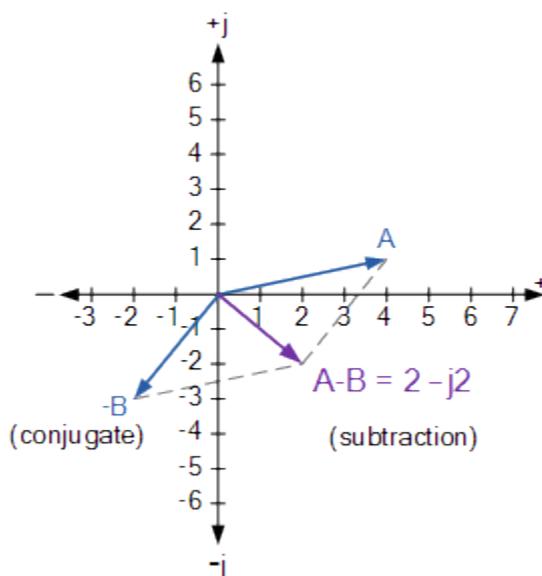
### Mathematical Subtraction

$$A - B = (4 + j1) - (2 + j3)$$

$$A - B = (4 - 2) + j(1 - 3)$$

$$A - B = 2 - j2$$

4

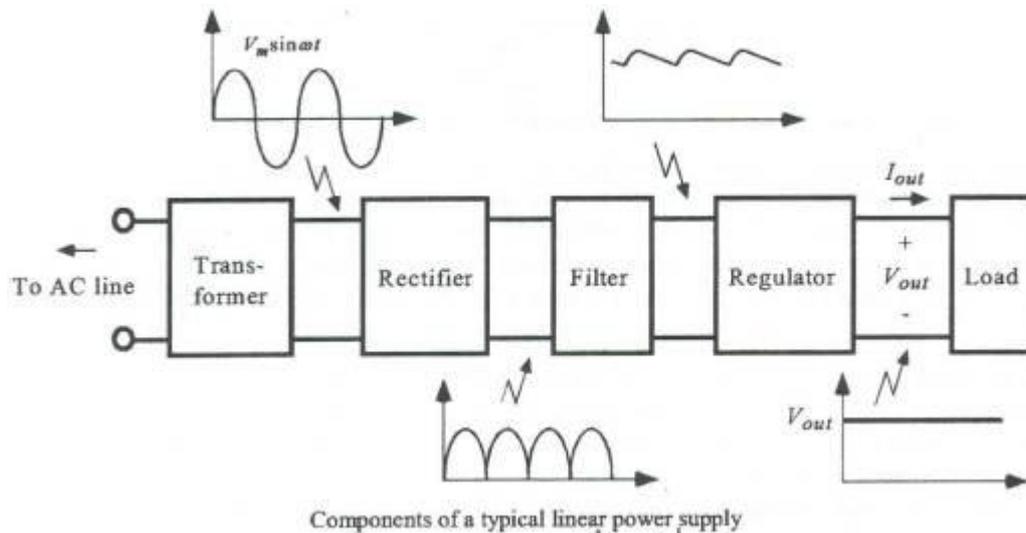


Reactance and impedance of pure R, L and C components

*See Handout for R, L and C components*

## Circuits

The block diagram for a stabilised power supply is shown below:



It can be seen that the power supply is broken down into the following sections:

- Transformer  
It performs two functions. It can step up or step down the a.c mains voltage. It also separates the dc supply circuit from the mains supply.
- Rectifier  
The rectifier converts ac voltage into dc voltage.
- Filter  
The filter smooth's the waveform from the rectifier to provide a true dc voltage with as little ripple as possible.
- Regulator  
The regulator is there to maintain the final required dc voltage independent of any changes in load current and mains voltage.
- Load  
The load is the device that requires the constant dc voltage.

These will now be discussed in more detail.

## The transformer

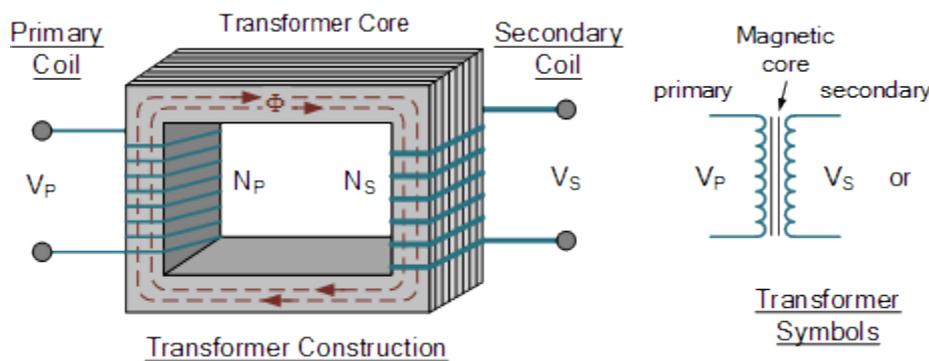
A transformer allows a high AC voltages and currents to be reduced to a much lower voltage level where it can be used in electrical circuits .It is an electronic component that uses two coils to interact where one coil induces a voltage on the second.

One coils of wire called the “Primary Winding” and another called the “Secondary Winding” are wound onto a core. The primary is connected to the high voltage and the secondary is normally connected to the circuit

These two coils are not in electrical contact with each other but are instead wrapped together around a common closed magnetic iron circuit the core. This soft iron core is not solid but made up of individual laminations connected together to help reduce the core’s losses.

The two coil windings are electrically isolated from each other but are magnetically linked through the common core allowing electrical power to be transferred from one coil to the other. When an electric current passed through the primary winding, a magnetic field is developed which induces a voltage into the secondary winding as shown.

### Transformer Construction (single-phase)



Where:

$V_P$  - is the Primary Voltage

$V_S$  - is the Secondary Voltage

$N_P$  - is the Number of Primary Windings

$N_S$  - is the Number of Secondary Windings

### A Transformers Turns Ratio

The number of turns on a transformer, and the supply voltage can determine the output voltage.

Consider the following equation;

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = n = \text{Turns Ratio}$$

---

A voltage transformer has 1500 turns of wire on its primary coil and 500 turns of wire for its secondary coil. What will be the turns ratio (TR) of the transformer.

2. If 240 volts rms is applied to the primary winding of the same transformer above, what will be the resulting secondary no load voltage.

## Power Diode Rectifier

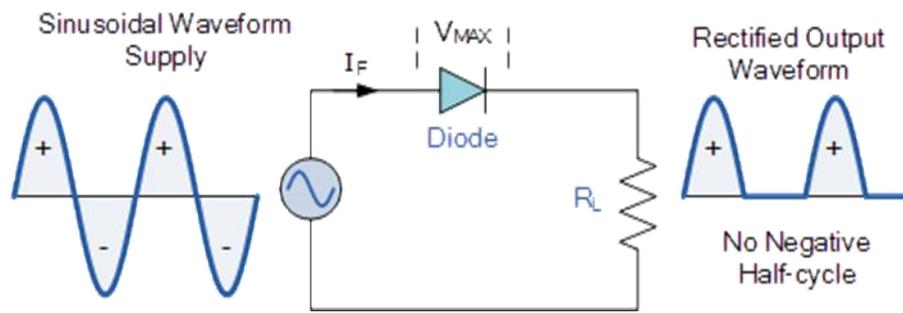
Power diodes can be used individually as above or connected together to produce a variety of rectifier circuits such as “Half-Wave”, “Full-Wave” or as “Bridge Rectifiers”

The most commonly used individual power diode for basic electronics applications is the general purpose 1N400x Series diode with standard ratings of continuous forward rectified current of 1.0 amp and reverse blocking voltage ratings from 50v for the 1N4001 up to 1000v for the 1N4007.

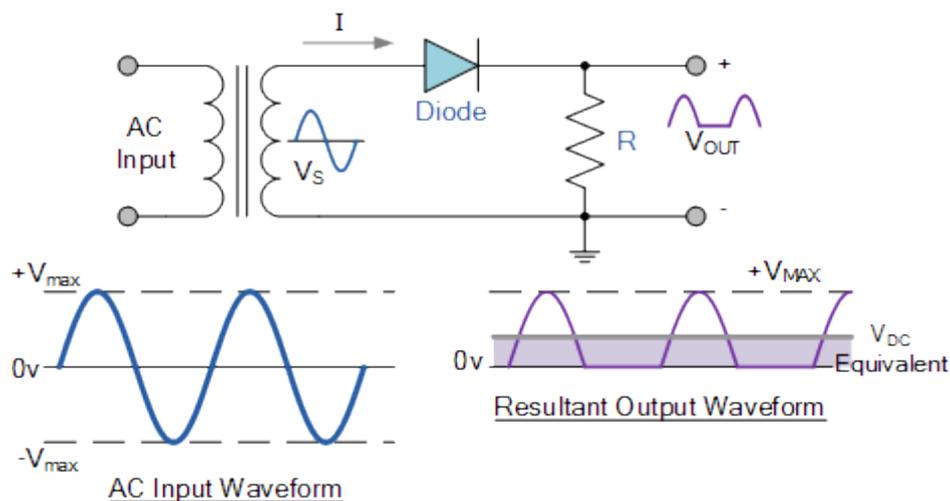
### Half Wave Rectification

A rectifier is a circuit which converts the *Alternating Current* (AC) input power into a *Direct Current* (DC) output power. The simplest of all the rectifier circuits being that of the **Half Wave Rectifier**.

The diode in a half wave rectifier circuit passes just one half of each complete sine wave of the AC supply in order to convert it into a DC supply. Then this type of circuit is called a “half-wave” rectifier because it passes only half of the incoming AC power supply as shown below.



### Half Wave Rectifier Circuit

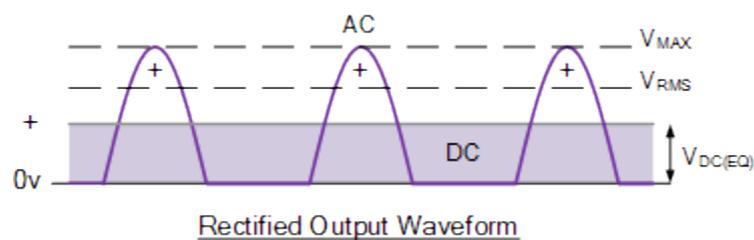


During each “positive” half cycle of the AC sine wave, the diode is *forward biased* as the anode is positive with respect to the cathode resulting in current flowing through the diode.

During each “negative” half cycle of the AC sinusoidal input waveform, the diode is *reverse biased* as the anode is negative with respect to the cathode. Therefore, NO current flows through the diode or circuit. Then in the negative half cycle of the supply, no current flows in the load resistor as no voltage appears across it so therefore,  $V_{out} = 0$ .

The current on the DC side of the circuit flows in one direction only making the circuit **Unidirectional**. As the load resistor receives from the diode a positive half of the waveform, zero volts, a positive half of the waveform, zero volts, etc, the value of this irregular voltage would be equal in value to an equivalent DC voltage of  $0.318 \times V_{max}$  of the input sinusoidal waveform or  $0.45 \times V_{rms}$  of the input sinusoidal waveform.

Then the equivalent DC voltage,  $V_{DC}$  across the load resistor is calculated as follows.



$$V_{d.c.} = \frac{V_{max}}{\pi} = 0.318V_{max} = 0.45V_S$$

Where  $V_{max}$  is the maximum or peak voltage value of the AC sinusoidal supply, and  $V_S$  is the RMS (Root Mean Squared) value of the supply.

During the rectification process the resultant output DC voltage and current are therefore both “ON” and “OFF” during every cycle. As the voltage across the load resistor is only present during the positive half of the cycle (50% of the input waveform), this results in a low average DC value being supplied to the load.

The variation of the rectified output waveform between this “ON” and “OFF” condition produces a waveform which has large amounts of “ripple” which is an undesirable feature. The resultant DC ripple has a frequency that is equal to that of the AC supply frequency.

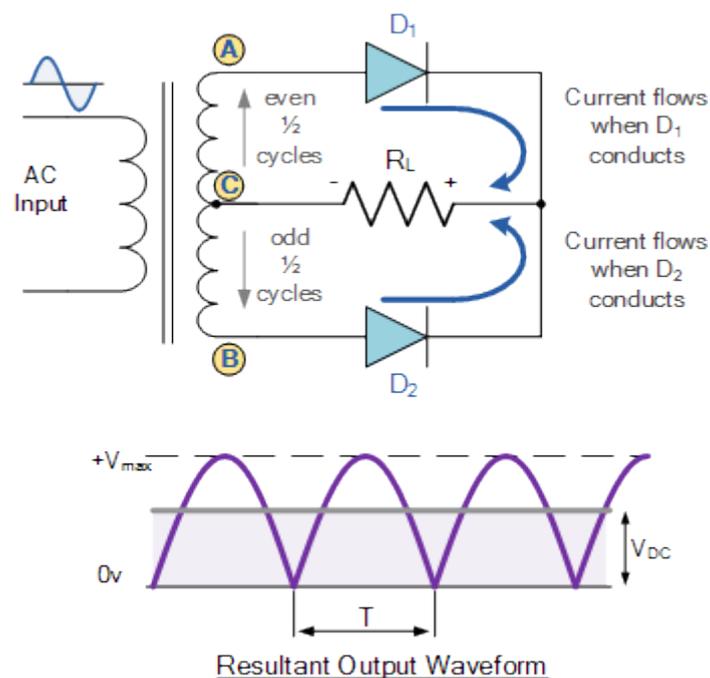
Very often when rectifying an alternating voltage we wish to produce a “steady” and continuous DC voltage free from any voltage variations or ripple. One way of doing this is to connect a large value Capacitor across the output voltage terminals in parallel with the load resistor as shown below. This type of capacitor is known commonly as a “Reservoir” or *Smoothing Capacitor* (This is covered later).

## The Full Wave Rectifier

The half wave rectifier circuit can be improved by using a full wave rectifier circuit. The average (DC) output voltage is higher than for half wave, the output of the full wave rectifier has much less ripple than that of the half wave rectifier producing a smoother output waveform.

In a Full Wave Rectifier circuit two diodes are now used, one for each half of the cycle. A multiple winding transformer is used whose secondary winding is split equally into two halves with a common centre tapped connection, (C). This configuration results in each diode conducting in turn when its anode terminal is positive with respect to the transformer centre point C producing an output during both half-cycles, twice that for the half wave rectifier so it is 100% efficient as shown below.

## Full Wave Rectifier Circuit



The full wave rectifier circuit consists of two *diodes* connected to a single load resistance ( $R_L$ ) with each diode taking it in turn to supply current to the load. When point A of the transformer is positive with respect to point C, diode  $D_1$  conducts in the forward direction as indicated by the arrows.

When point B is positive (in the negative half of the cycle) with respect to point C, diode  $D_2$  conducts in the forward direction and the current flowing through resistor R is in the same direction for both half-cycles.

As the spaces between each half-wave developed by each diode is now being filled in by the other diode the average DC output voltage across the load resistor is now double that of the

single half-wave rectifier circuit and is about  $0.637V_{\max}$  of the peak voltage, assuming no losses.

$$V_{d.c.} = \frac{2V_{\max}}{\pi} = 0.637V_{\max} = 0.9V_{RMS}$$

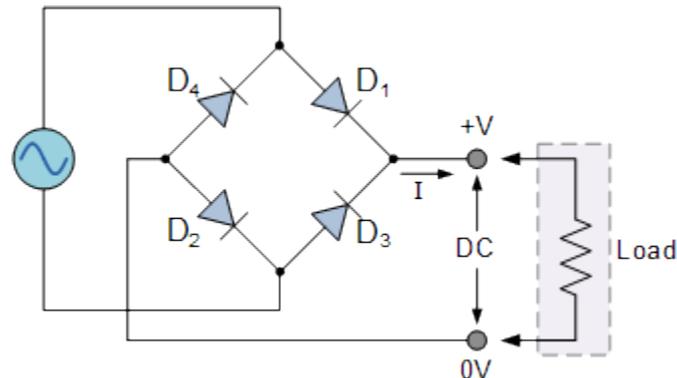
Where:  $V_{\max}$  is the maximum peak value in one half of the secondary winding and  $V_{RMS}$  is the rms value.

The peak voltage of the output waveform is the same as before for the half-wave rectifier provided each half of the transformer windings have the same rms voltage value. To obtain a different DC voltage output different transformer ratios can be used. The main disadvantage of this type of full wave rectifier circuit is that a larger transformer for a given power output is required with two separate but identical secondary windings making this type of full wave rectifying circuit costly compared to the "Full Wave Bridge Rectifier" circuit equivalent.

## The Full Wave Bridge Rectifier

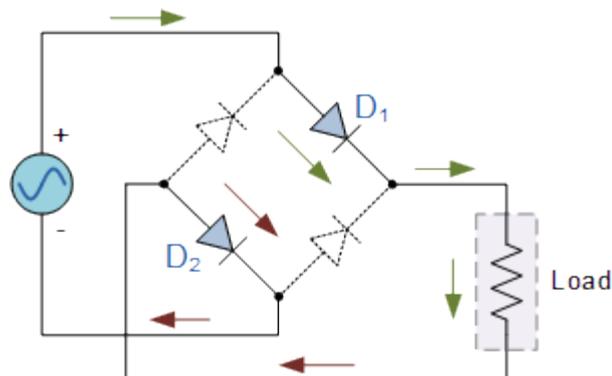
Another type of circuit that produces the same output waveform as the full wave rectifier circuit, is that of the **Full Wave Bridge Rectifier**. This type of single phase rectifier uses four individual rectifying diodes connected in a closed loop “bridge” configuration to produce the desired output. The main advantage of this bridge circuit is that it does not require a special centre tapped transformer, thereby reducing its size and cost. The single secondary winding is connected to one side of the diode bridge network and the load to the other side as shown below.

The Diode Bridge Rectifier



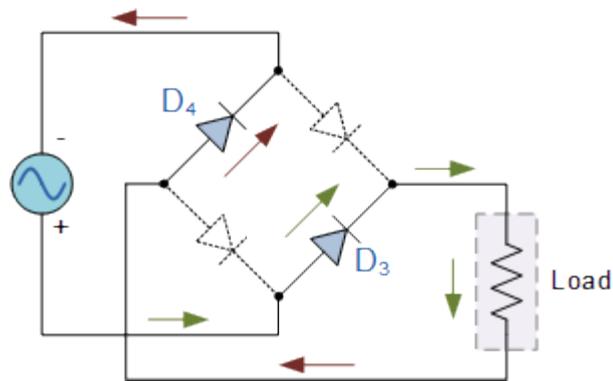
The four diodes labelled D<sub>1</sub> to D<sub>4</sub> are arranged in “series pairs” with only two diodes conducting current during each half cycle. During the positive half cycle of the supply, diodes D<sub>1</sub> and D<sub>2</sub> conduct in series while diodes D<sub>3</sub> and D<sub>4</sub> are reverse biased and the current flows through the load as shown below.

The Positive Half-cycle



During the negative half cycle of the supply, diodes D<sub>3</sub> and D<sub>4</sub> conduct in series, but diodes D<sub>1</sub> and D<sub>2</sub> switch “OFF” as they are now reverse biased. The current flowing through the load is the same direction as before.

## The Negative Half-cycle

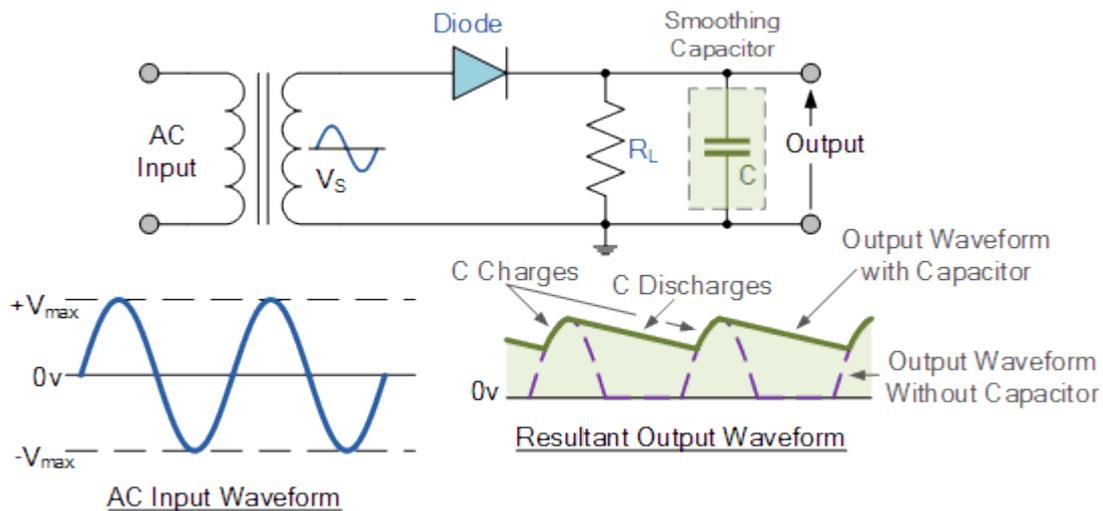


As the current flowing through the load is unidirectional, so the voltage developed across the load is also unidirectional the same as for the previous two diode full-wave rectifier, therefore the average DC voltage across the load is  $0.637V_{\max}$ .



## The Filter

### Half-wave Rectifier with Smoothing Capacitor



When rectification is used to provide a direct voltage ( DC ) power supply from an alternating ( AC ) source, the amount of ripple voltage can be further reduced by using larger value capacitors but there are limits both on cost and size to the types of smoothing capacitors used.

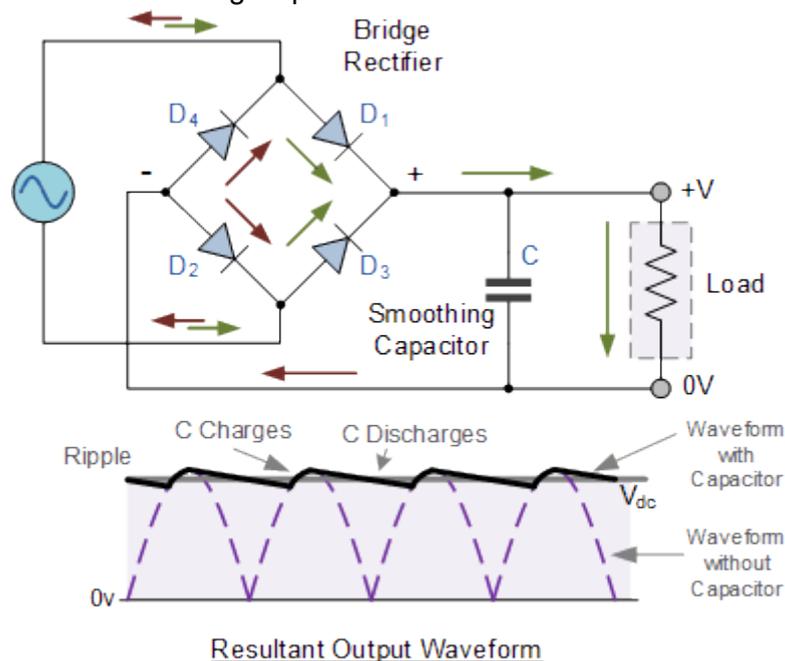
For a given capacitor value, a greater load current (smaller load resistance) will discharge the capacitor more quickly ( **RC Time Constant** ) and so increases the ripple obtained. Then for single phase, half-wave rectifier circuit using a power diode it is not very practical to try and reduce the ripple voltage by capacitor smoothing alone. In this instance it would be more practical to use "Full-wave Rectification" instead.

In practice, the half-wave rectifier is used most often in low-power applications because of their major disadvantages being. The output amplitude is less than the input amplitude, there is no output during the negative half cycle so half the power is wasted and the output is pulsed DC resulting in excessive ripple.

## The Smoothing Capacitor

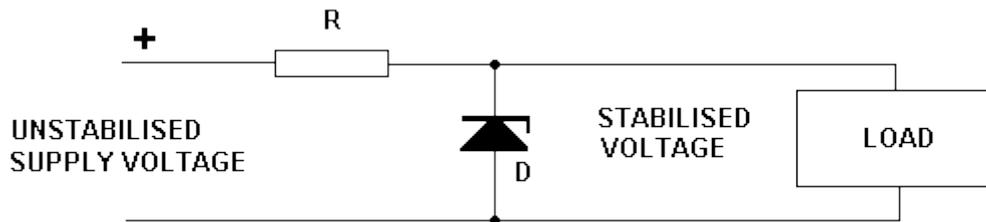
The single phase half-wave rectifier produces an output wave every half cycle and that it was not practical to use this type of circuit to produce a steady DC supply. The full-wave bridge rectifier however, gives us a greater mean DC value ( $0.637 V_{max}$ ) with less superimposed ripple while the output waveform is twice that of the frequency of the input supply frequency. We can therefore increase its average DC output level even higher by connecting a suitable smoothing capacitor across the output of the bridge circuit as shown below.

### Full-wave Rectifier with Smoothing Capacitor



The smoothing capacitor converts the full-wave rippled output of the rectifier into a smooth DC output voltage. Generally for DC power supply circuits the smoothing capacitor is an Aluminium Electrolytic type that has a capacitance value of  $100\mu\text{F}$  or more with repeated DC voltage pulses from the rectifier charging up the capacitor to peak voltage.

## The Regulator



The Zener diode is operated in reverse bias mode (positive on its cathode). It relies on the reverse breakdown voltage occurring at a specified value. This value is printed on it. It has two main applications.

1. as a reference source, where the voltage across it is compared with another voltage.
2. as a voltage regulator, smoothing out any voltages variations occurring in the supply voltage across the load.

When being used a voltage regulator, if the voltage across the load tries to rise then the Zener takes more current. The increase in current through the resistor causes an increase in voltage dropped across the resistor. This increase in voltage across the resistor causes the voltage across the load to remain at its correct value.

In a similar manner, if the voltage across the load tries to fall, then the Zener takes less current. The current through the resistor and the voltage across the resistor both fall. The voltage across the load remains at its correct value.