CAPACITORS AND CAPACITANCE

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2 Understand the concepts of capacitance and determine capacitance values in DC circuits

What is a Capacitor?

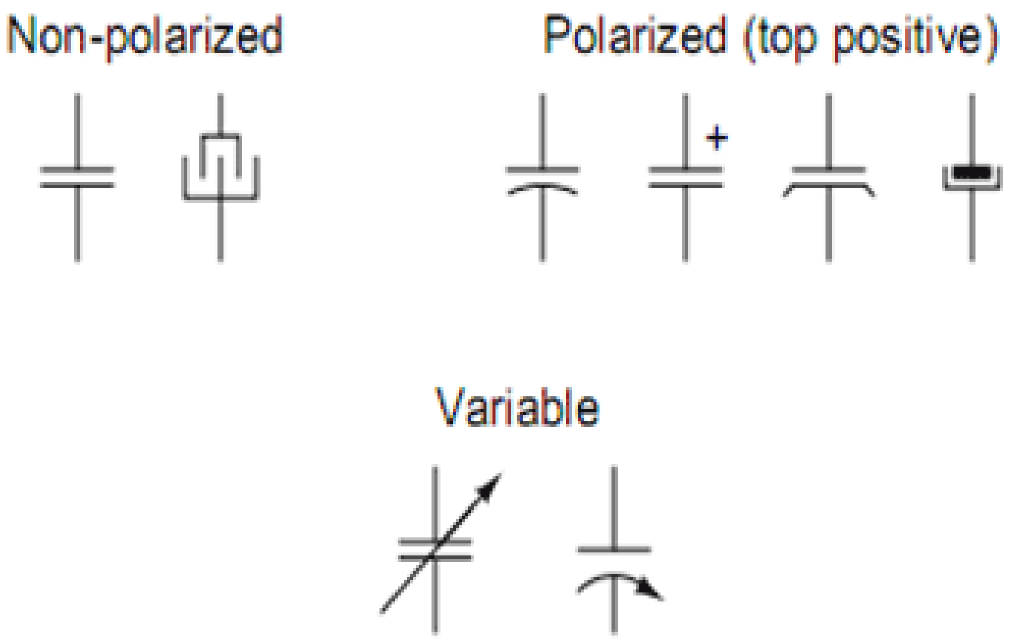
A **capacitor** is an electronic component that stores electric charge. The charge is a unit of electricity called the **Coulomb (Q)**. One Coulomb flows past a given point of a circuit when a current of one ampere is maintained for one second.

$$Q = IT$$

It consists of two conductors which are separated by a dielectric medium. The potential difference when applied across the conductors polarizes the ions to store the charge in the dielectric medium.

The symbol of a capacitor is:

There are more specialised symbols available.



The capacitance or the potential storage by the capacitor is measured in **Farads** which is symbolized as ‘F’. One Farad is the capacitance when one coulomb of electric charge is stored in the conductor on the application of one volt potential difference.

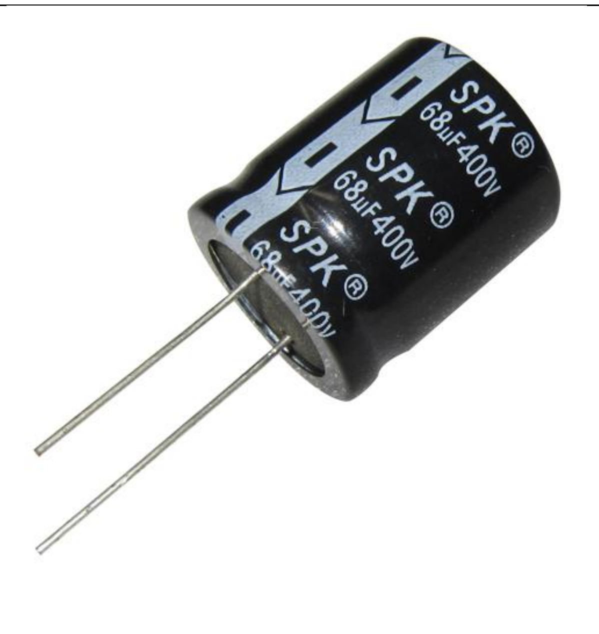
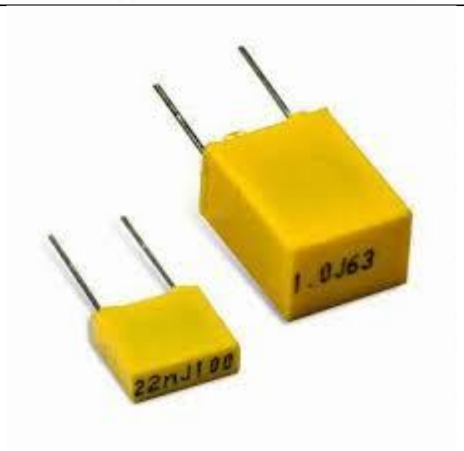
The charge stored in a capacitor is given by

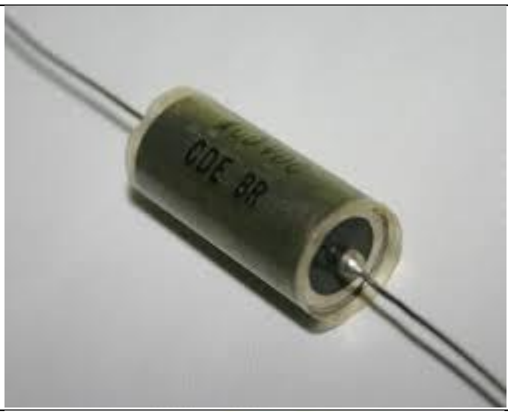
$$Q = CV$$

Where

Q - Charge stored by the capacitor
C - Capacitance value of the capacitor
V - Voltage applied across the capacitor

Capacitors Types

		Typical Values	Permittivity
Fixed			
Electrolytic			Relative Permittivity **εr =**
Mica			Relative Permittivity **εr =** 4 to 7
Plastic			Relative Permittivity **εr =**

Paper			Relative Permittivity **εr =** 2 to 3
Ceramic		1nF and 1µF, although values up to 100µF are possible	Relative Permittivity **εr =** 5 to 10000
Variable			
			

Research activity:

(i) Complete the table above indicating typical capacitance values

(ii) (ii) Produce a report explaining the Construction of the capacitors in the table above indicating the dielectric material used.

Capacitor Uses

Capacitors are widely used in a variety of applications of electronic circuits such as:

- storing a charge such as in a camera flash circuit
- smoothing in a power supply
- coupling between stages of circuits (they block DC voltages)
- filter circuits
- delay functions
- tuning radios to particular frequencies
- phase alteration.

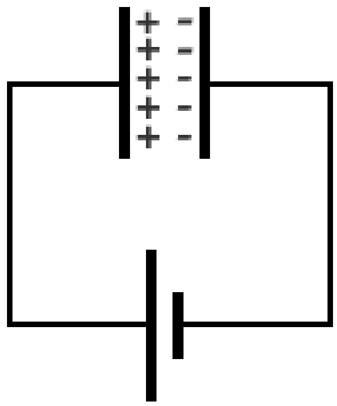
:

Task: Generate a report explaining the above applications

Capacitors and capacitance

An excess of electrons on a body constitutes an electric charge, with the unit of charge being the Coulomb.

Consider two metal plates situated in free space as shown.



In SI units, one line of electric flux is assumed to leave a positive charge of 1 Coulomb and to enter a negative charge of 1 Coulomb.

Electric flux = **Ψ**, = Q Coulomb's

Electric flux density D = Q/a Coulombs / M^2

Electric field strength, E = V/d Volts/Metre

The ratio of Electric flux density / Electric field strength = D/E is known as the

PERMITTIVITY OF FREE SPACE, symbol **εo** and in SI units has a value of

8.85 x 10 $^{-12}$ Farads / metre.

The unit of capacitance is the Farad. A capacitor has a capacitance of 1 Farad if it holds a charge of 1 Coulomb when the applied potential is 1V.

From this equation: Charge Q = CV Coulombs.

Capacitance between plates

Electric Field Strength, E

Electric field strength, $E = \frac{V}{d}$ volts/metre

V = Supply Voltage (Volt, V)
d = Distance between plates (meter, m)
A = Area (meter^2, m^2)

*Electric field strength is also called **potential gradient.**

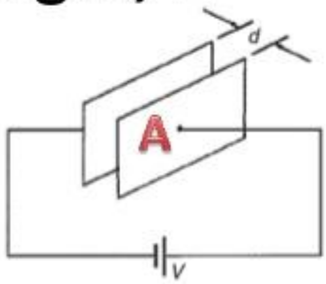


Figure 6.3

Electric Flux Density, D

Electric flux density D *is the amount of flux passing* through a defined area *A that is perpendicular to the* direction of the flux

electric flux density, $D = \frac{Q}{A}$ coulombs/metre^2

Q = Charges (coulombs, C)
A = Area (meter^2, m^2)

*Electric flux density is also called **charge density, σ.**

Consider two parallel metal plates, each having an area of a^2 metres and a separated distance of d meters in free space.

Permittivity of free space **εo** $= \frac{D}{E}$ $= \frac{\text{Electric flux density}}{\text{Electric field strength}}$

$$= \frac{\frac{Q}{a}}{\frac{V}{d}} \qquad \text{but } Q = CV$$

$$= \frac{\frac{C\cancel{V}1}{a}}{\cancel{V}}$$

d

$$\boldsymbol{\varepsilon o} = \frac{\dfrac{C}{a}}{\dfrac{1}{d}}$$

$$\frac{\boldsymbol{\varepsilon o}}{d} = \frac{C}{a}$$

$$\mathbf{C} = \frac{\boldsymbol{\varepsilon o}\ \mathbf{a}}{\mathbf{d}}$$

Task: Analyse the derivation of the above equation

If the space between the plates are filled with an insulating material (known as a dielectric), the capacitance is increased. The factor by which the capacitance is increased is known as the **RELATIVE PERMITTIVITY**, symbol **εr** of the dielectric.

The product **εr εo** is known as the **ABSOLUTE PERMITTIVITY**, symbol **ε** of the dielectric.

For a capacitor having a material dielectric between the plates,

Capacitance C = εo εr a /d farads

Material	Relative Permittivity **εr**
Air	1.006
Wax impregnated paper	2 to 3
Glass	5 to 10
Mica	4 to 7
Ceramic	5 to 10000
Polythene	2.
Aluminium oxide	8.7

Example 1

A parallel plate capacitor consists of two metal plates, each 600 x 500mm spaced 1mm apart. The dielectric between the plates is mica having a relative permittivity of 6 and a p.d of 3000v is applied between the plates.

Calculate

(i) The capacitance
(ii) The electric flux density
(iii) The electric field strength in the dielectric.

Permittivity of free space, **εo** = 8.85×10^{-12} Farads / metre.

(i) Capacitance, C = **εo εr a /d**
$C = 8.85 \times 10^{-12} \times 6 \times 0.3 / 0.001$
$C = 1.593 \times 10^{-8}$
C = 15.9 nF

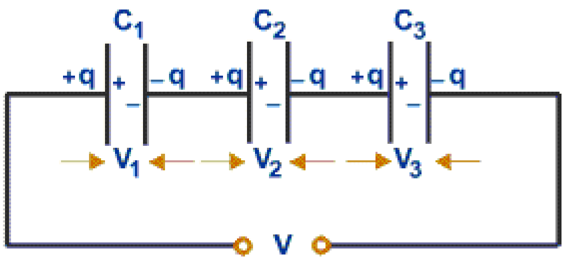
(ii) Charge Q = CV
$Q = 15.93 \times 10^{-9} \times 3000$
$Q = 47.8 \times 10^{-6}$ Coulombs
Q = 47.8 **µ**C

Flux density, D = Q/a
$D = 47.8 \times 10^{-6} / 600 \times 500$mm 10^{-6}
$D = 159\ 10^{-6}$ Coulombs/m^2
D = 159 **µ**C / m^2

(iii) Electric field strength E = V/d
$E = 3000 / 1 \times 10^{-3}$
3×10^{6} V/m
= 3 Mv / m

Attempt: Worksheet C1

Capacitors connected in series



V1

When the supply is first switched on, the current must flow into each capacitor and since it flows for the same length of time the capicitors must have the same charge.(Q = It).

Let the charge on each of the capacitors be Q Coulombs.

Then V1 = Q/C1, V2 = Q/C2, V3 = Q/C3

V1 = V1 + V2 + V3

Now consider the three capacitors to be replaced by a single capicitor, C_T Farads which will hold a charge of Q Coulombs at a p.d of V Volts.

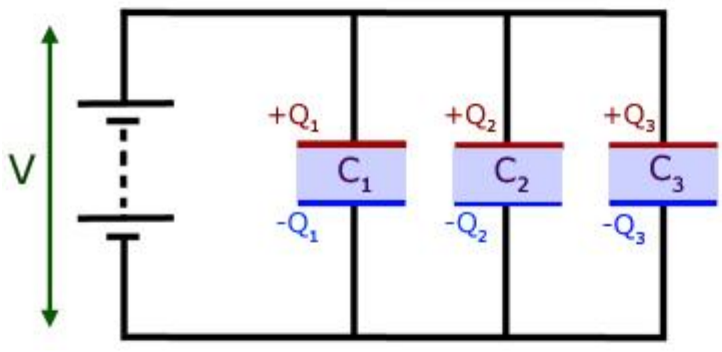
$V = Q/ C_T$

Substitute V for V1, V2, V3 we get

$Q/ C_T = Q/C1 + Q/C2 + Q/C3$

$1/ C_T = 1/C1 + 1/C2 + 1/C3$

Capacitors connected in parallel



Charge on Capacitor C1 = Q1

Charge on Capacitor C1 = C1 V Coulombs

Charge on Capacitor C2 = Q2

Charge on Capacitor C2 = C2 V Coulombs

Charge on Capacitor C3 = Q3

Charge on Capacitor C3 = C3 V Coulombs

Total charge taken from the supply Q is Q1 + Q2 + Q3

Now consider the three capacitors to to be replaced by a single capicitor , C_T Farads which will hold a charge of Q Coulombs at a p.d of V Volts.

Then $Q = C_T V$ Coulombs

Substitute Q, Q1, Q2, Q3

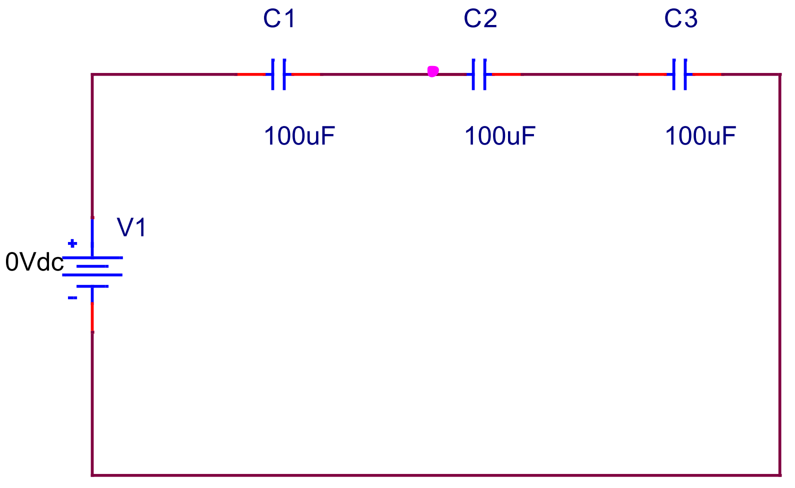
we get

$C_T V = C1 V + C2V + C3V$

$C_T = C1 + C2 + C3$

Example 1

Calculate the capacitance for the following circuit.



$1/C_T = 1/C1 + 1/C2 + 1/C3$

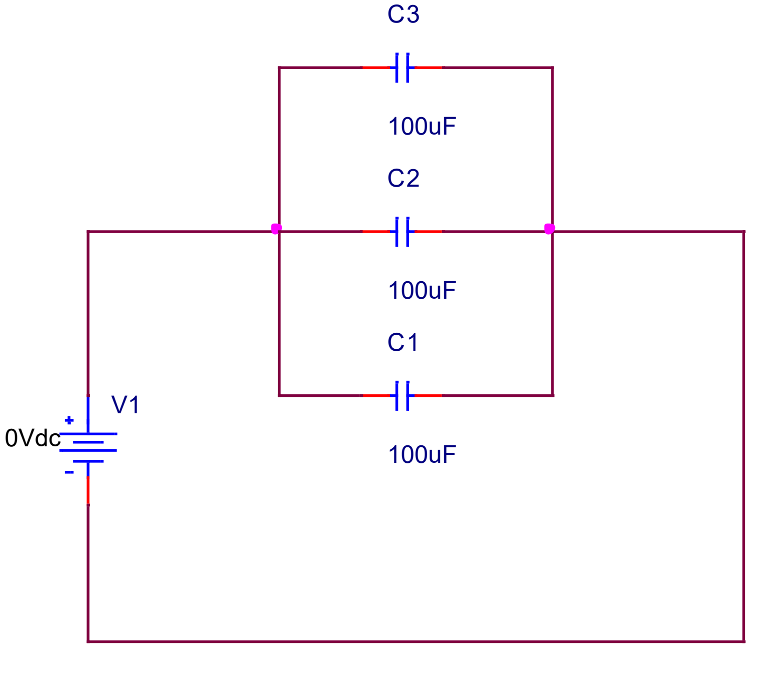
$1/C_T = 1/100 \times 10^{-6} + 1/100 \times 10^{-6} + 1/100 \times 10^{-6}$

$1/C_T = 30000$

$C_T = 33.33uF$

Example 2

Calculate the capacitance for the following circuit.



$C_T = C1 + C2 + C3$

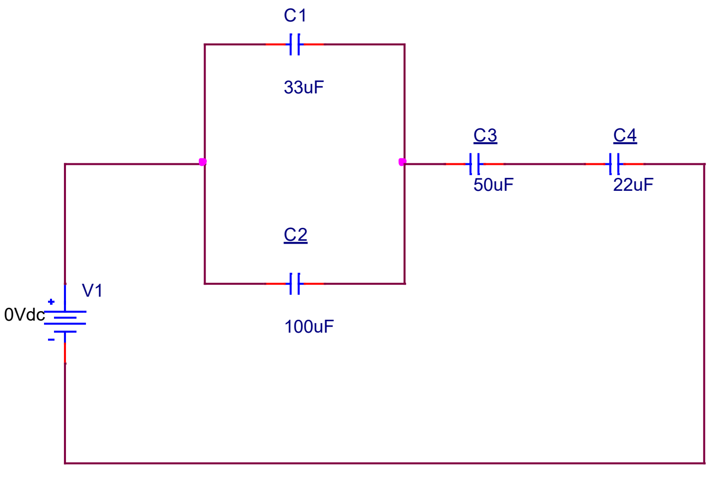
$C_T = 100 \times 10^{-6} + 100 \times 10^{-6} + 100 \times 10^{-6}$

$C_T = 300 \times 10^{-6}$

$C_T = 300uF$

Example 3

Calculate the capacitance for the following circuit.



Parallel Capacitance Network

$C_T = C1 + C2$

$C_T = 33 \times 10^{-6} + 100 \times 10^{-6}$

$C_T = 133 \times 10^{-6}$

$C_T = 133uF$

Series Capacitance Network

$1/ C_T = 1/C_P + 1/C3 + 1/C4$

$1/ C_T = 1/133 \times 10^{-6} + 1/50 \times 10^{-6} + 1/22 \times 10^{-6}$

$1/ C_T =$

$C_T =$ uF

Attempt: Worksheet C2

Energy stored in a capacitor

Suppose that a capacitor is charged at a constant current until it reaches a p.d of V volts. Since the current is constant, the increase in charge per second is constant and hence the rate of increase of voltage is constant.

The average voltage across the capacitor is V/2. Since the current is constant, the energy supplied to the capacitor is (V/2) x It.

But It is the charge on the capacitor as mentioned previously and Q = CV.

Energy, W = V/2 x CV

Energy, W = 1/2 x CV^2

Example

If it is required to store 1.25 Joules of energy in a 100 **μ**F capacitor. Calculate the voltage to which the capacitor must be charged.

$W = 1/2 \times CV^2$

$1.25 = 1/2 \times 100 \times 10^{-6}\ V^2$

$V^2 = 1.25 / 0.5\ \times 100 \times 10^{-6}$

$V = \sqrt{25000}$

V = 158 Volts

Attempt: Worksheet C3

Charging a Capacitor

When a supply is connected to a capacitor via a series resistor, the initial current is high as the supply transports the charge from one plate of the capacitor to the other. This charging current approaches zero as the capacitor becomes fully charged up to the supply voltage. Charging the capacitor stores energy in the electric field between the capacitor plates.

The rate of charging is typically described in terms of a time constant, τ =RC.

At τ, the capacitor will be charged up to 0.636 the supply voltage

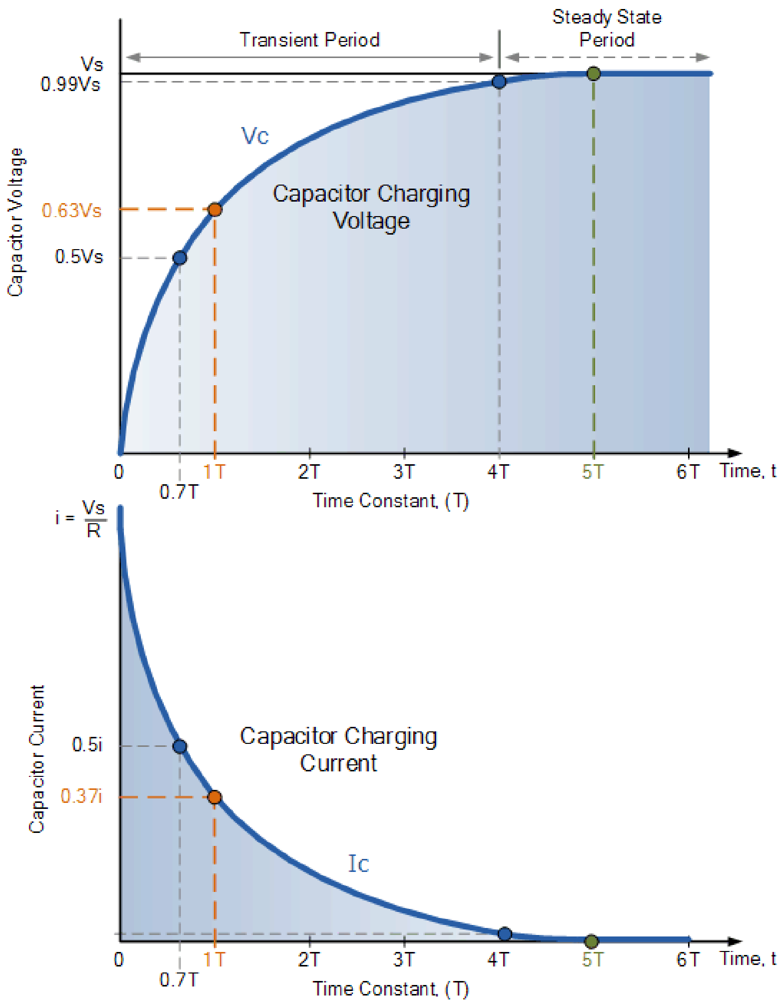
The Voltage across the capacitor is

$V_C = V_S(1 - e^{-t/RC})$

$I_C = (V_S/R)\, e^{-t/RC}$

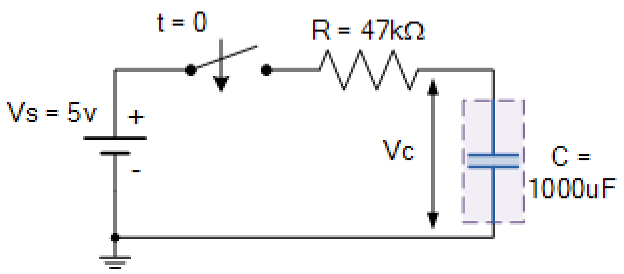
It will take 5 τ to fully charge the capacitor.

Capacitor charging waveforms



Example

Calculate the time constant of the following circuit.



The time constant τ is found using the formula T = R x C in seconds.

Therefore the time constant τ is:

T = R x C = 47k x 1000uF = 47 Secs

a) What value will be the voltage across the capacitor at 0.7 time constants?

At 0.7 time constants (0.7T) Vc = 0.5Vs. Therefore, Vc = 0.5 x 5V = 2.5V

b) What value will be the voltage across the capacitor at 1 time constant?

At 1 time constant (1T) Vc = 0.63Vs. Therefore, Vc = 0.63 x 5V = 3.15V

c) How long will it take to "fully charge" the capacitor?

The capacitor will be fully charged at 5 time constants.

1 time constant (1T) = 47 seconds, (from above). Therefore, 5T = 5 x 47 = 235 secs

d) The voltage across the Capacitor after 100 seconds?

The voltage formula is given as Vc = V(1 - $e^{-t/RC}$)

which equals: Vc = 5(1-$e^{-100/47}$) RC = 47 seconds from above, Therefore, Vc = 4.4 volts

Discharging a Capacitor

If a capacitor has been fully charged and the supply is removed and replaced by a short circuit the capacitor will discharge via the resistor.

In a **RC Discharging Circuit**, the time constant (τ)is still equal to 63%. Then for a RC discharging circuit that is initially fully charged, the voltage across the capacitor after one time constant, 1T, has dropped to 63%of its initial value which is 1 - 0.63 = 0.37 or 37% of its final value.

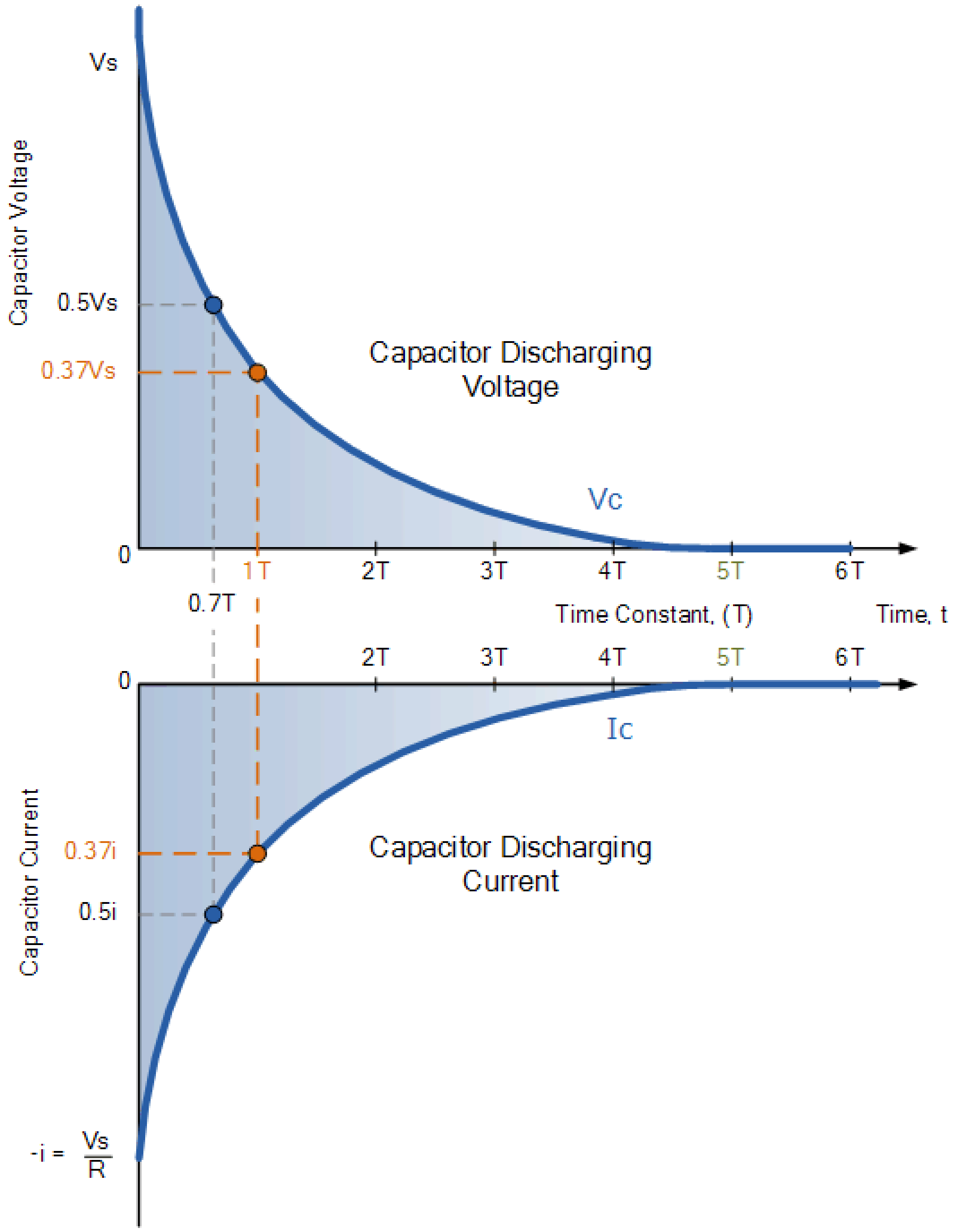
So now this is given as the time taken for the capacitor to discharge down to within 37%of its fully discharged value which will be zero volts (fully discharged), and in our curve this is given as0.37Vc.

As the capacitor discharges, it loses its charge at a declining rate. At the start of discharge the initial conditions of the circuit, are t = 0, i = 0 and q = Q. The voltage across the capacitors plates is equal to the supply voltage and Vc = Vs. As the voltage across the plates is at its highest value maximum discharge current flows around the circuit.

$V_c = V_s\ e^{-t/RC}$

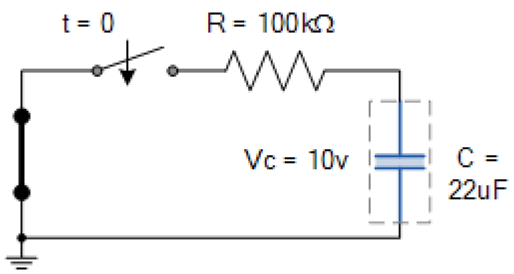
$I_c = I_o\ e^{-t/CR}$ where $I_o = V_o / R$

Discharing waveforms



Example

Calculate the time constant of the following RC discharging circuit.



The time constant, T of the circuit is found using the following formula T = R x C given in seconds.

Therefore, the RC circuits time constant T is:

T = R x C = 100k x 22uF = 2.2 Seconds

a) What value will be the voltage across the capacitor after 1 time constant?

At 1 time constant (1T) Vc = 0.37Vc. Therefore, Vc = 0.37 x 10V = 3.7V

c) How long will it take for the capacitor to "fully discharge" itself (5 time constants)?

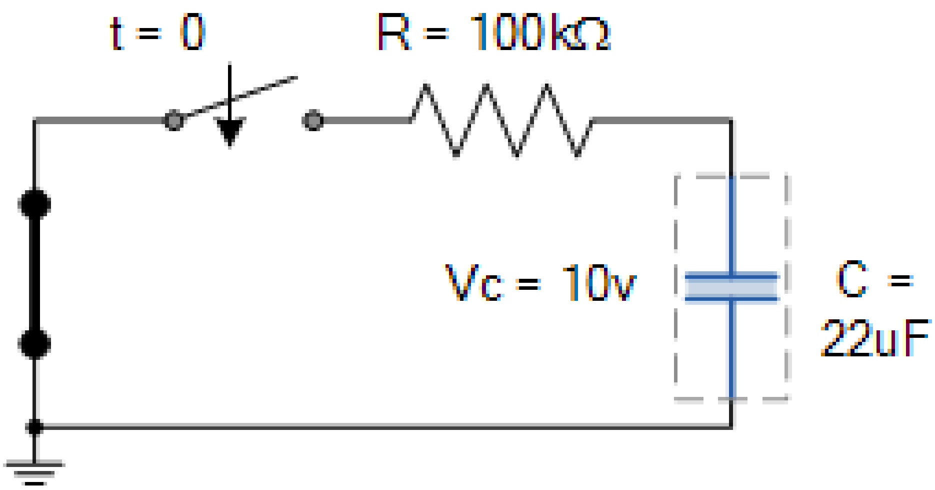
1 time constant (1T) = 2.2 seconds. Therefore, 5T = 5 x 2.2 = 11 Seconds

Charging a capacitor

Consider the following equations for the circuit shown

$Vc = Vs\ (1 - e^{-t/RC})$

$IC = (Vs/R)\ e^{-t/RC}$



Complete the following table

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
Vc (v)																					

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
Ic (A)																					

RC Time constants

The time required to charge a capacitor to 63.2 percent of full charge or to discharge it to 36.8 percent of its initial voltage is known as the TIME CONSTANT (TC) of the circuit.

The value of the time constant in seconds is equal to the product of the circuit resistance in ohms and the circuit capacitance in farads.

The value of one time constant is expressed mathematically as $t = RC$.

E.g. What is the time constant of a series RC circuit that contains a 1 megohm resistor and a 10-microfarad capacitor?

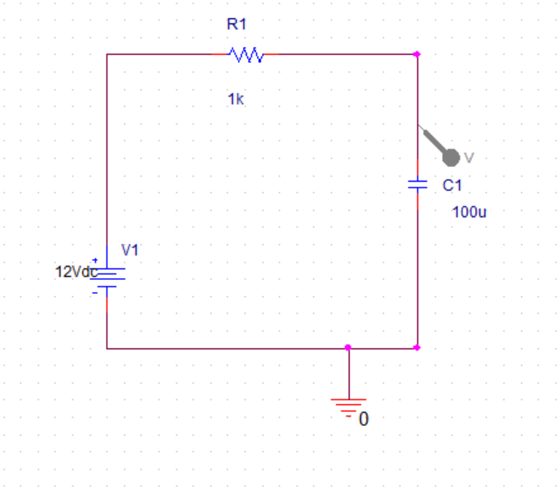
$T = RC$

$T = 1 \times 10^{6} \times 10 \times 10^{-6} = 10$ seconds

Attempt: Worksheet C3

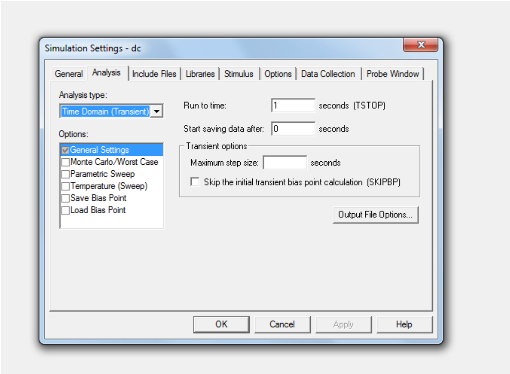
Pspice: Capacitor Charging

Circuit

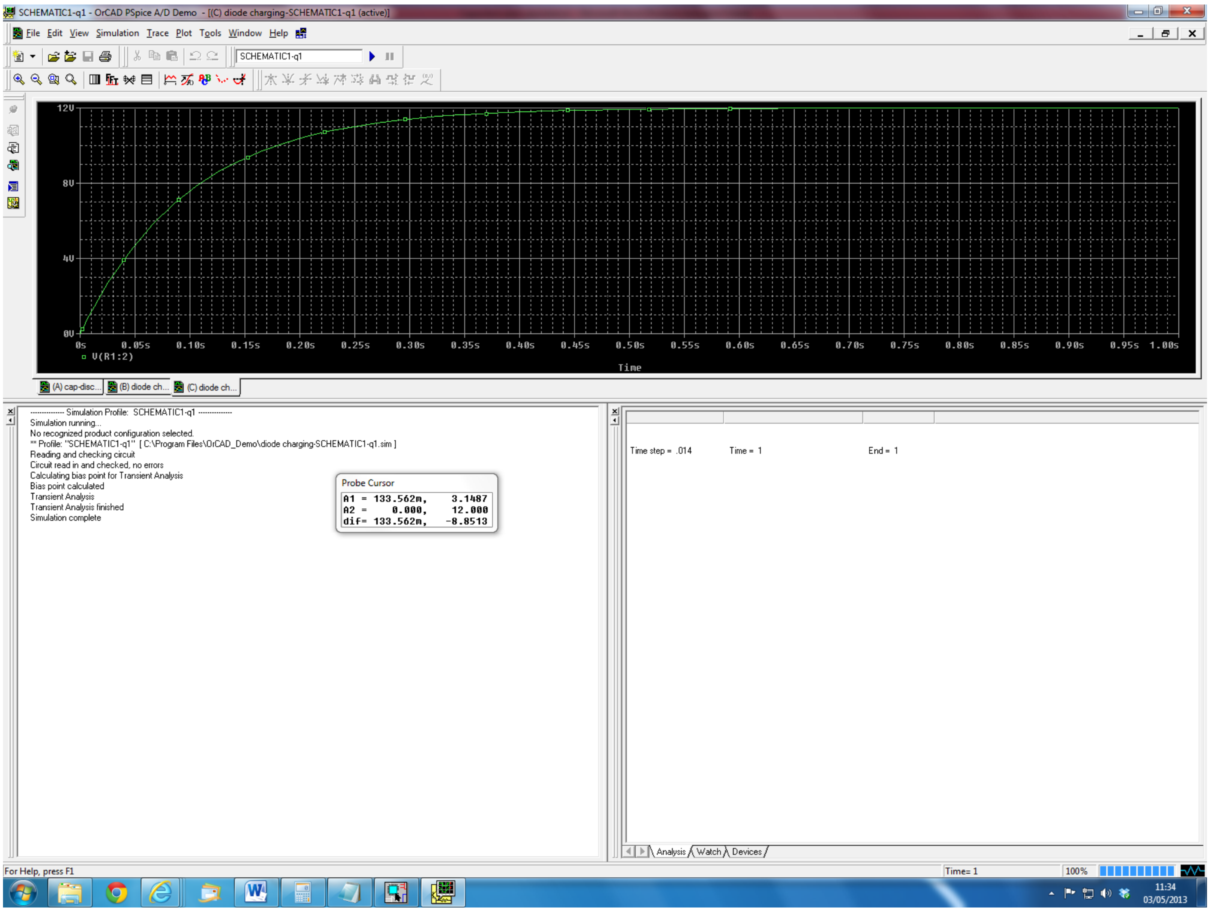


Ensure that you set the initial Charge of the capacitor to 0v by doubling clicking on the capacitor.

Analysis setting

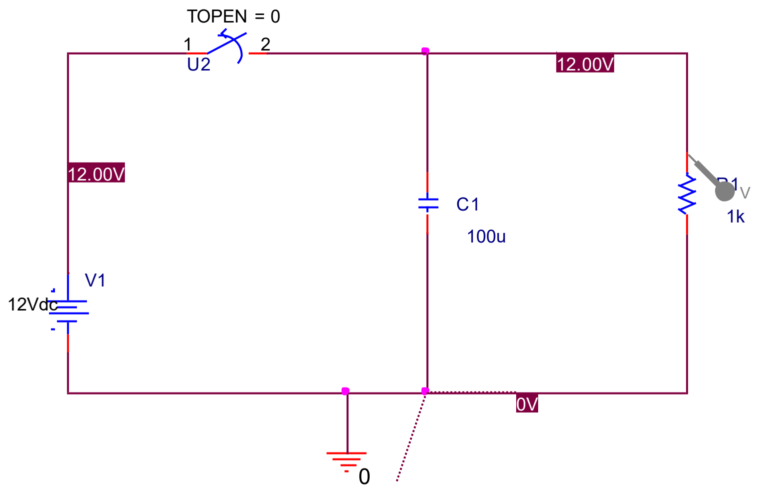


Expected output

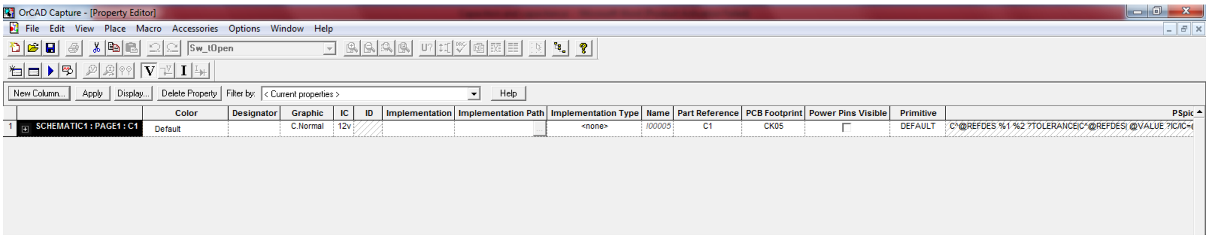


Pspice: Capacitor discharging

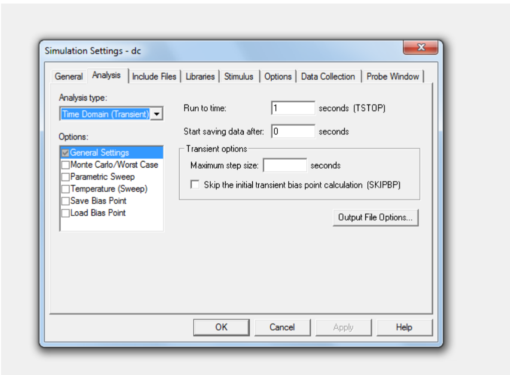
Circuit



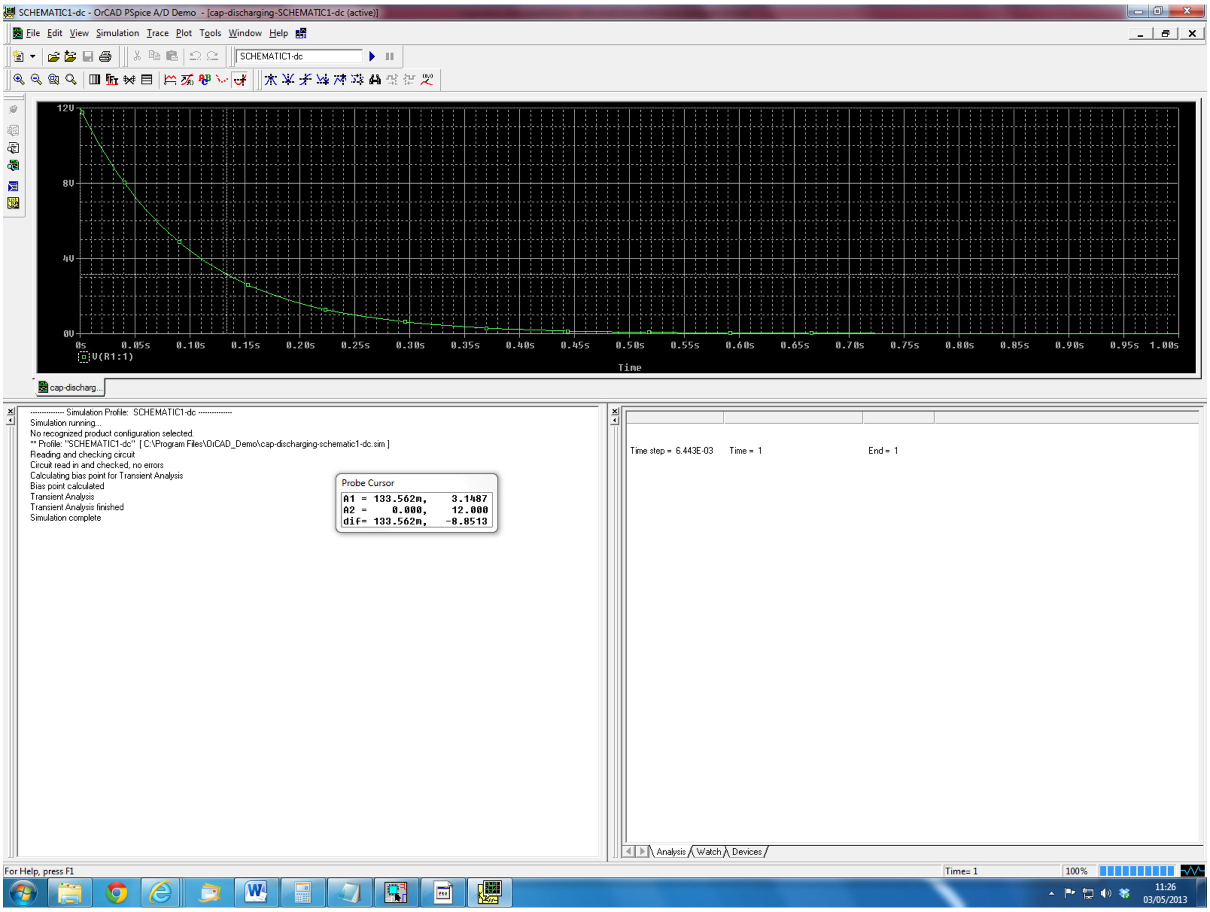
Ensure that you set the initial Charge of the capacitor to 12v by doubling clicking on the capacitor.



Analysis setting



Expected output



Practical: Charging a capacitor

Equipment

DC Power supply (0 to 30v)

Two large electrolytic capacitors, 10μF

Two 1 MΩ resistors

One Breadboard

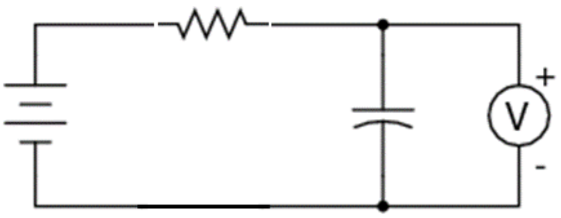
One DMM

Note: Ensure you identify the polarity on the capacitor. Usually capacitors of the size specified have a negative (-) marking toward the negative terminal. Failure to heed proper polarity will almost surely result in capacitor failure. When electrolytic capacitors fail, they typically explode, spewing caustic chemicals and emitting foul odours.

LEARNING AIMS

Capacitor charging action
Capacitor discharging action
Time constant calculation

SCHEMATIC DIAGRAM



Charging circuit

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INSTRUCTIONS

1. Build the "charging" circuit on the breadboard. Get your circuit checked before turning on your power supply.

2. Set the power supply to 30V. Do not enable the power supply.

3. Enable the power supply and measure the voltage across the capacitor every 5 seconds.

4. Complete the table in Appendix A.

5. The "time constant" (τ) of a resistor capacitor circuit is calculated by taking the circuit resistance and multiplying it by the circuit capacitance. This is the amount of time it takes for the capacitor voltage to increase approximately 63.2% from its present value to its final value: the voltage of the battery.

6. Turn off the power supply and record the voltage across the capacitor every 5 seconds using the table in Appendix A.

7. Plot the voltage of a charging/discharging capacitor over time on a sheet of graph paper.

8. Repeat with a second capacitor.

9. Indicate the time constant on the graph and see if the expected voltage is obtain.

10. Generate a lab report for this experiment using the following:

- Aim
- Method
- Results
- Conclusion

Appendix A

Time	Charging		Discharging
Time	Voltage (V)		Voltage (V)
0			
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			
55			
60			